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Coupled charged Bose quantum wires

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Abstract. We investigate the effect of many-body correlations on the ground-state properties of two coupled charged Bose quantum wires. The intrawire and interwire correlations are treated on the same footing within the self-consistent mean-field theory of Singwi, Tosi, Land and Sjölander. Static properties, *viz.*, pair-correlation functions, local-field correction factors, screened interactions and susceptibilities, are calculated for a range of wire spacings d, wire radii R_0, and boson density parameters r_s. We find that the qualitative dependence of intrawire correlations on r_s and R_0 remains the same as found for an isolated wire, except that they become slightly weaker with the decreasing spacing d. The interwire correlations, on the other hand, depend strongly on d and are found to grow in magnitude with decreasing d. Further, we find no evidence for the existence of a charge-density-wave ground state in the density range investigated, $1 \leqslant r_s \leqslant 8$, in the close proximity of wires. A comparison with the similar studies on the coupled electron quantum wires reveals that the charge-density-wave instability observed there may be an artifact of the neglect of interwire correlation effects.

1. Introduction

There has been considerable recent interest in the study of layered charged quantum liquids of both Fermi (electron or hole) and Bose types. The former, i.e., the layered electron systems, can be fabricated at the interfaces of semiconductor heterostructures with fine control on the system parameters due to the advent of nanotechnology such as computerized molecular beam epitaxy. These electron systems also occur naturally in copper oxide planes present in ceramic superconductors. The charged Bose systems, on the other hand, have not been so far realized in the laboratory, but their study has received much recent theoretical [1, 2] importance mainly due to their recognition as a possible model for understanding the phenomenon of high-T_c superconductivity. Due to this close resemblance to superconductors and to the observation of some unusual and interesting phenomena, the study of ground-state behaviour of both the layered electron and charged Bose systems has emerged in itself as an important problem. A variety of new features are shown to appear due entirely to the presence of other layers of particles. The possibility of charge-density-wave (CDW) ground states and the enhancement of Wigner crystallization density [3, 4] are among some typical examples.

Existence or non-existence of CDW instability in these structures has remained a controversial issue in the last five years. Controversy arises mainly between the results of Neilson and co-workers [3] and Kalman and co-workers [5]. The theoretical procedures of the two groups differ in the method of treatment of many-body correlation effects. Very recently, the difference in results for the electron system has been somewhat narrowed down with an important contribution by Liu *et al* [6]. It is shown in agreement with the conclusions of

Kalman *et al* that the possibility of a CDW ground state diminishes with the proper treatment of many-body correlations.

Parallel to the study of layered structures, the possibility of CDW ground states has also been investigated in the double-quantum-wire electron system [7, 8]. Interestingly, in addition to the finite-wavevector ($q = 2q_F$) instability, a long-wavelength instability has been shown to appear at a wire separation lower than that for instability at $q = 2q_F$. Here, q_F is the one-dimensional (1D) Fermi wavevector. In a quantum wire system, the particles are dynamically free only along one spatial direction, while their motion is quantum mechanically restricted in the remaining two transverse directions. With the existing technology, it is now possible to fabricate these electron systems in the laboratory. In the present work, we investigate a related problem of a coupled charged Bose quantum wire system consisting of two parallel and identical quasi-1D (Q1D) charged Bose systems. Study of this system, when compared with its Fermi counterpart, may be useful to disentangle the statistical and particle contributions to the many-body correlations. Our first motive is to study the static structure of the system, and secondly we intend to examine whether the transition to the CDW ground state occurs as predicted in the electron system. A simple cylindrical model proposed by Gold and Ghazali [9] in the context of electron wires is used where analytical expressions for the intra- and interwire Coulomb potentials have been developed in Fourier space. We deal with the many-body correlations among charged bosons within the self-consistent theory of Singwi, Tosi, Land and Sjölander (STLS) [10] as generalized to the double-wire system. In the STLS approach the effect of correlations is represented by static local-field corrections to the bare interactions between the particles. The local fields are determined numerically in a self-consistent way. It is worthwhile to mention here that the STLS theory has proved quite successful in treating the correlation effects in 3D [11] and low-dimensional [12] systems provided the particle number density is not too low and its importance beyond the random-phase approximation (RPA) has recently been shown by us for the single charged Bose quantum wire [13]. Static properties we wish to calculate include the static pair-correlation functions, local-field correction factors and static screened interaction potentials. We also discuss in a systematic way the importance of correlations in calculating these properties for a range of model parameters which comprise the boson number density, the wire radius and the wire spacing. To highlight the role of interwire correlations, results are compared with the lower-order calculation where these correlations are simply neglected and with the results of an isolated single wire.

The paper is organized as follows. In section 2 we first present the wire model and then outline the STLS theory for the double-wire system. Results and discussion are given in section 3. Section 4 contains discussion on the possibility of a CDW ground state. In section 5 we make some concluding remarks.

2. Wire model and theoretical formalism

2.1. Wire model

We consider two parallel cylindrical quantum wires in the x-direction, with wire radius R_0 and with infinite potential barriers at $|\boldsymbol{r}| = R_0$. The wires are assumed to be separated by a perpendicular distance $d \geqslant 2R_0$. With these assumptions, the wavefunctions of particles in two wires do not overlap and therefore, the tunnelling of particles between wires is not possible in the present model. We take the density of bosons n in each wire to be equal. The motion of the carriers is free along the cylinder axis, while it is restricted perpendicular to the cylinder. At absolute zero temperature, the bosons are assumed to be present in the condensate state. The wire system is assumed to be embedded in a uniform neutralizing background. For the

Coulomb interaction potentials between bosons, we use the analytical results developed by Gold and Ghazali [7, 9]. The intra- and interwire potentials are given, respectively, as

$$V_{11}(q) = \frac{e^2}{2\epsilon_0} f_{11}(q) \tag{1}$$

and

$$V_{12}(q) = \frac{e^2}{2\epsilon_0} f_{12}(q) \tag{2}$$

where $f_{11}(q)$ and $f_{12}(q)$ are the intra- and interwire form factors given, respectively, by

$$f_{11}(q) = \frac{144}{(qR_0)^2}\left[\frac{1}{10} - \frac{2}{3(qR_0)^2} + \frac{32}{3(qR_0)^4} - 64\frac{I_3(qR_0)K_3(qR_0)}{(qR_0)^4}\right] \tag{3}$$

$$f_{12}(q) = (96)^2\left[\frac{I_3(qR_0)}{(qR_0)^3}\right]^2 K_0(qd). \tag{4}$$

$I_n(x)$ and $K_n(x)$ are the modified Bessel functions of order n. ϵ_0 is the dielectric constant of the background and we will use $\epsilon_0 = 1$. In the long wavelength limit both $f_{11}(q)$ and $f_{12}(q)$ exhibit logarithmic divergence and the limiting results are given by

$$f_{11}(q) = -4\left[\ln(qR_0/2) + C - 73/120 + \mathrm{O}(q)\right] \tag{5}$$

and

$$f_{12}(q) = -4\left[\ln(qd/2) + C + \mathrm{O}(q)\right]. \tag{6}$$

$C = 0.577$ is the Euler constant. These limiting results are useful while performing the numerical calculations.

2.2. *Theoretical formalism*

Within the linear response framework, the density response function for the double-wire system can be expressed in the form of a 2×2 matrix given as

$$\left[\chi_{ij}(q,\omega)\right]^{-1} = \begin{bmatrix} \chi_1^{-1}(q,\omega) & -V_{12}(q)(1 - G_{12}(q)) \\ -V_{21}(q)(1 - G_{21}(q)) & \chi_2^{-1}(q,\omega) \end{bmatrix} \tag{7}$$

where $\chi_i\ (q,\omega)$ $(i = 1, 2)$ is the density response function for the single wire. In the STLS approximation, $\chi_i(q,\omega)$ is given by

$$\chi_i(q,\omega) = \frac{\chi_i^0(q,\omega)}{1 - V_{ii}(q)\left[1 - G_{ii}(q)\right]\chi_i^0(q,\omega)} \qquad i = 1, 2 \tag{8}$$

where $\chi_i^0(q,\omega)$ is the response function for the noninteracting charged Bose system, and at absolute zero it is given by

$$\chi_i^0(q,\omega) = \frac{2n_i\epsilon_q}{\left[(\omega + \mathrm{i}\eta)^2 - \epsilon_q^2\right]}. \tag{9}$$

$\epsilon_q = \overline{h}^2q^2/2m$ is the free particle energy and η is a positive infinitesimal quantity. As $n_1 = n_2$, we have $\chi_1^0(q,\omega) = \chi_2^0(q,\omega)$. In equations (7) and (8), $G_{ij}(q)$ $(i, j = 1, 2)$ are the static local-field correction factors that account for the short-range Coulomb correlation effects. Within the STLS approach the local fields are related to the static structure factors $S_{ij}(q)$ through the expression

$$G_{ij}(q) = -\frac{1}{n}\int_{-\infty}^{\infty}\frac{\mathrm{d}k}{2\pi}\frac{kV_{ij}(k)}{qV_{ij}(q)}\left[S_{ij}(q-k) - \delta_{ij}\right]. \tag{10}$$

The fluctuation-dissipation theorem relates $S_{ij}(q)$ to $\chi_{ij}(q,\omega)$ as

$$S_{ij}(q) = -\frac{\hbar}{\pi n}\int_0^\infty \mathrm{d}\omega\,\chi_{ij}(q,\mathrm{i}\omega) \tag{11}$$

where the frequency integration is to be performed along the imaginary axis. In view of the geometry of the system, we have $A_{ij}(q) = A_{ji}(q)$, $i = j = 1, 2$, where A may be S or G. From equations (7), (10) and (11), it is apparent that $G_{ij}(q)$ is to be obtained numerically in a self-consistent way.

3. Results and discussion

In the numerical calculations and the results presented (if otherwise mentioned) we choose a system of units in which $\hbar = 1$ and lengths and energies are expressed, respectively, in units of the Bohr atomic radius (a_0) and the Rydberg (1 Rydberg $= e^2/(2a_0)$). The density of bosons is described by a dimensionless parameter r_s; $r_s = 1/(2na_0)$.

3.1. Static correlation functions

Equations (7), (10) and (11) are solved numerically for the intra- and interwire correlation functions in a self-consistent way within a tolerance of 0.001%. For very thin and closely spaced wires, it becomes extremely difficult to obtain the convergent solution. In these situations, we employ a numerical procedure which makes use of local fields given by

$$G_{ij}(q) = \frac{1}{2}\left[\frac{G_{ij}^{m-1}(q) + G_{ij}^{m-2}(q)}{2} + G_{ij}^{m}(q)\right] \tag{12}$$

in the mth iteration for the calculation of $S_{ij}(q)$. The same method is used for calculating $G_{ij}(q)$ from $S_{ij}(q)$. In this way the convergent solutions are obtainable for $d \geqslant 1.1(2R_0)$ when R_0 is small and, equivalently, if r_s is large. Results for the intra- and interwire structure factors $S_{11}(q)$ and $S_{12}(q)$ and their corresponding local fields $G_{11}(q)$ and $G_{12}(q)$ are plotted, respectively, in figures 1 and 2 for $r_s = 1$, 3, 5, 8 and $R_0 = 2$. For $r_s = 8$, it becomes almost impossible to obtain the accurate self-consistent solution for $d < 4.45$ and the curve for $r_s = 8$ corresponds to $d = 4.45$, while $d = 4$ for other r_s values. Solid and dashed lines represent, respectively, the intra- and interwire quantities. The interwire structure factor $S_{12}(q)$ is about an order of magnitude smaller than $S_{11}(q)$ and remains negative in the range of q-values of interest. Relative dependence of intra- and interwire correlations on r_s is more clear in the behaviour of local fields (figure 2). It is apparent that both $G_{11}(q)$ and $G_{12}(q)$ grow in strength with decreasing density and $G_{12}(q)$ remains always comparatively weaker than $G_{11}(q)$.

In figures 3 and 4 we show, respectively, the dependence of two local fields on the wire spacing d and the wire size R_0. Curves are labelled the same as in figure 1. $G(q)$ for the single wire is shown for comparison as a dash–dot line for $r_s = 8$. It is interesting to note that as the wires are brought closer the intrawire local field does not undergo any significant change, while the interwire local field is affected strongly. In fact $G_{11}(q)$ becomes somewhat weaker as compared to the single wire and for $d \geqslant 8$ its behaviour does not differ considerably from $G(q)$ of single wire. On the other hand, the interwire local field grows continuously with decreasing spacing. In figure 3, our results are for $r_s = 8$; $R_0 = 2$ and $d = 8$, 6, 4.45. Figure 4 displays the R_0-dependence of two local fields for $r_s = 5$ and $d = 4$. $G_{11}(q)$ increases with the decreasing wire size and if compared with its d-dependence, the R_0-dependence is rather stronger. $G_{12}(q)$ depends upon R_0 in the same manner as $G_{11}(q)$ except at small q. For other

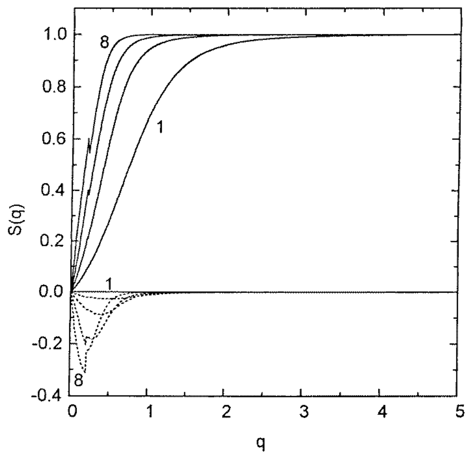


Figure 1. The intrawire and interwire static structure factors $S_{11}(q)$ (solid lines) and $S_{12}(q)$ (dashed lines) for $r_s = 1$, 3 and 5; $d = 4$ and $r_s = 8$; $d = 4.45$. $R_0 = 2$ for all the curves. Labels denote the boson number density r_s. Curves for $S_{11}(q)$ from bottom to top correspond to $r_s = 1$, 3, 5 and 8, while for $S_{12}(q)$ the sequence of curves is exactly the reverse of $S_{11}(a)$.

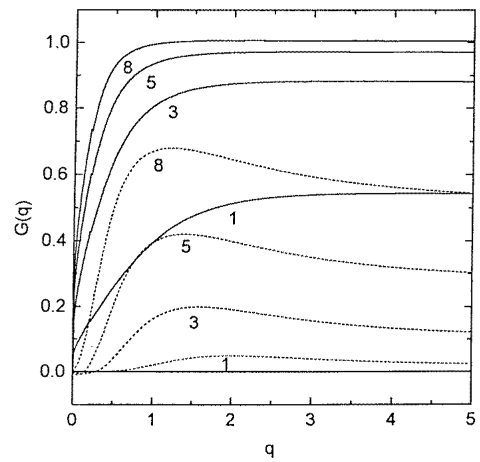


Figure 2. The intrawire and interwire local-field factors $G_{11}(q)$ (solid lines) and $G_{12}(a)$ (dashed lines). Model parameters are the same as in figure 1. Labels denote the boson number density r_s.

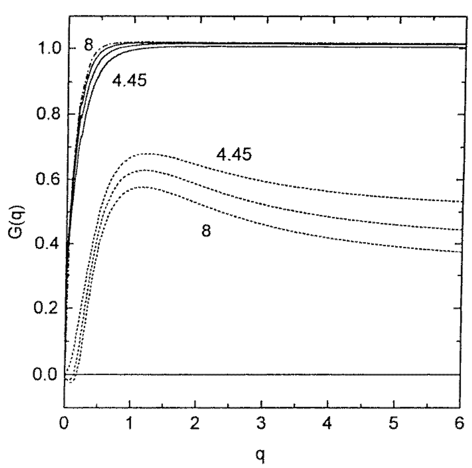


Figure 3. The intrawire and interwire local-field factors $G_{11}(q)$ (solid lines) and $G_{12}(q)$ (dashed lines) for $r_s = 8$ and $R_0 = 2$. Curves for $G_{11}(q)$ from top to bottom correspond to $d = 8$, 6 and 4.45, while for $G_{12}(q)$ the sequence of curves is exactly the reverse of $G_{11}(q)$. The dash–dot line is the local-field factor of an isolated single wire at $r_s = 8$.

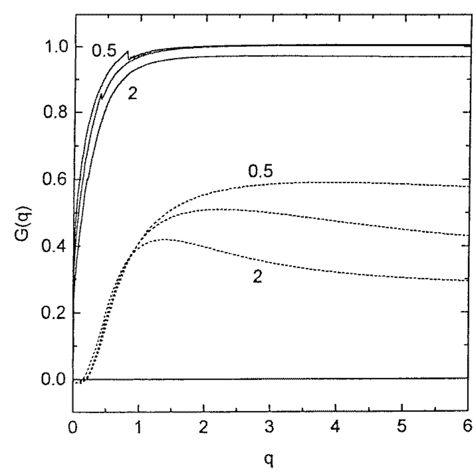


Figure 4. The intrawire and interwire local-field factors $G_{11}(q)$ (solid lines) and $G_{12}(q)$ (dashed lines) for $r_s = 5$ and $d = 4$. Curves for both $G_{11}(q)$ and $G_{12}(q)$ from top to bottom correspond to $R_0 = 0.5$, 1 and 2.

r_s-values, we have found qualitatively similar d–R_0-dependence. Thus, in the limit of narrow and closely spaced wires, the interwire correlations become quite significant and are, therefore, expected to influence the properties of the system containing their effect.

In the long-wavelength limit, i.e., $q \to 0$, $G_{ij}(q)$ behaves approximately as

$$G_{ij}(q) \approx -\frac{2r_s}{\pi f_{ij}(q)} \int_0^\infty \mathrm{d}k \left[S_{ij}(k) - \delta_{ij}\right]\left[f_{ij}(k) + k f'_{ij}(k)\right] \tag{13}$$

where f' denotes the first-order derivative of f w.r.t. k. It is interesting to note that the factor $[G_{ij}(q) f_{ij}(q)]$ becomes merely independent of q in the small-q limit and its value depends only on r_s, R_0 and d. This limiting expression for $G_{ij}(q)$, as we will see in section 4, plays a crucial role in determining the possibility of existence of a long-wavelength CDW instability.

Results for the static pair-correlation functions $g_{ij}(x)$, which can be obtained from the inverse Fourier transform of $S_{ij}(q)$ as

$$g_{ij}(x) = 1 + \frac{2r_s}{\pi} \int_0^\infty \mathrm{d}k \cos(kx) \left[S_{ij}(k) - \delta_{ij}\right] \tag{14}$$

are plotted in figure 5. $g_{ii}(x)$ defines, as usual, the probability that the two particles in wire i are separated by a relative distance of x, while $g_{ij}(x)$ represents the probability of finding a particle in wire j at a distance x (parallel to the wire i) given that there is a particle present at the origin ($x = 0$) in the wire i. Figures 5(a) and 5(b) contain, respectively, the dependence of $g_{11}(x)$ and $g_{12}(x)$ on r_s for fixed R_0 and d. For $r_s = 1$, $g_{12}(x)$ is everywhere close to unity and thereby implies weak interwire correlations. With increasing r_s, however, the interwire correlations are seen to become stronger. For example, by $r_s = 8$, $g_{12}(x = 0) = 0.56$. Since $x = 0$ for $g_{12}(x)$ corresponds to particles at the origin in both the wires separated by a perpendicular distance d ($d = 4.45$ in this case) this value of $g_{12}(0)$ defines strong interwire correlations. Also shown for comparison in figure 5(a) is $g(x)$ for the single wire for $r_s = 8$ as a dash–dot line. Both $g_{11}(x)$ and $g(x)$ are small and negative in the limit of small separation. Since negative $g(x)$ corresponds to an unphysical situation it is not justified to use the STLS approximation for treating short-range correlations for $r_s \geqslant 8$ for $R_0 = 2$. With decreasing R_0, the negative region in $g_{11}(x)$ appears already at low r_s-values. Thus, the domain of validity of the STLS approximation is decided together by r_s and R_0.

In figures 5(c) and 5(d), we compare, respectively, $g_{11}(x)$ and $g_{12}(x)$ for $r_s = 8$ and $R_0 = 2$ for the different wire spacings. As observed in the behaviour of local fields (figure 3) the intrawire correlations show only weak dependence on d, while the interwire correlations are strongly affected.

3.2. Static screened pair potentials

Proceeding in the way described by us for the double-layer system [4], the real space expressions for the static screened intra- and interwire potentials are obtained, respectively, as

$$V_{11}^{sc}(x) = \frac{1}{\pi} \int_0^\infty \mathrm{d}k \cos(kx) \Bigg\{ f_{11}(k) + \frac{\dot\chi(k,0)}{1 - \dot\chi^2(k,0) f_{12}^2(k)(1 - G_{12}(k))^2} \times \left[f_{11}^2(k) + f_{12}^2(k) + 2\dot\chi(k,0)(1 - G_{12}(k)) f_{11}(k) f_{12}^2(k)\right] \Bigg\} \tag{15}$$

and

$$V_{12}^{sc}(x) = \frac{1}{\pi} \int_0^\infty \mathrm{d}k \cos(kx) \Bigg\{ f_{12}(k) + \frac{\dot\chi(k,0)}{1 - \dot\chi^2(k,0) f_{12}^2(k)(1 - G_{12}(k))^2} \times \left[2 f_{11}(k) f_{12}(k) + \dot\chi(k,0)(1 - G_{12}(k)) f_{12}^3(k) + f_{11}^2(k) f_{12}(k)\right] \Bigg\} \tag{16}$$

where we have used the notation, $e^2\chi_1(k,0) = e^2\chi_2(k,0) = \dot\chi(k,0)$. Figures 6(a) and 6(b) show, respectively, the dependence of $V_{11}^{sc}(x)$ and $V_{12}^{sc}(x)$ on r_s for fixed R_0 and d (solid lines).

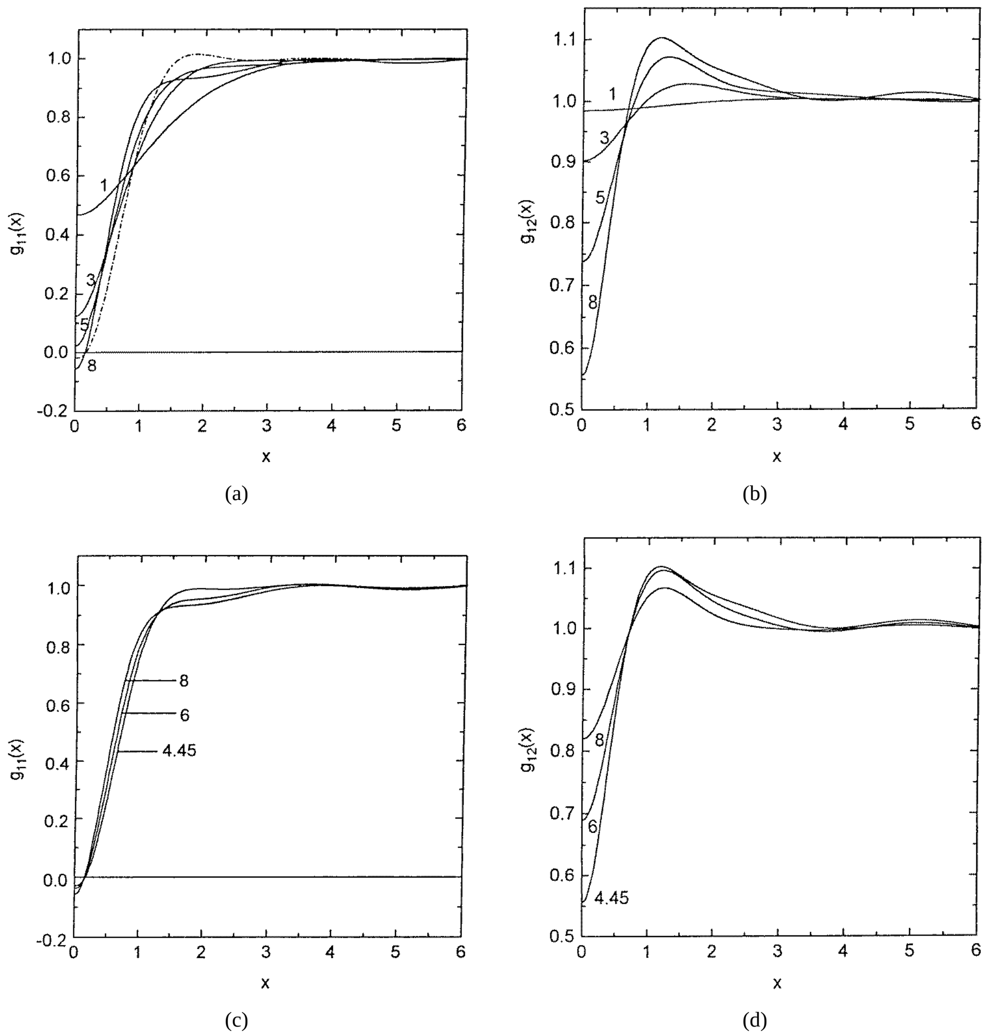
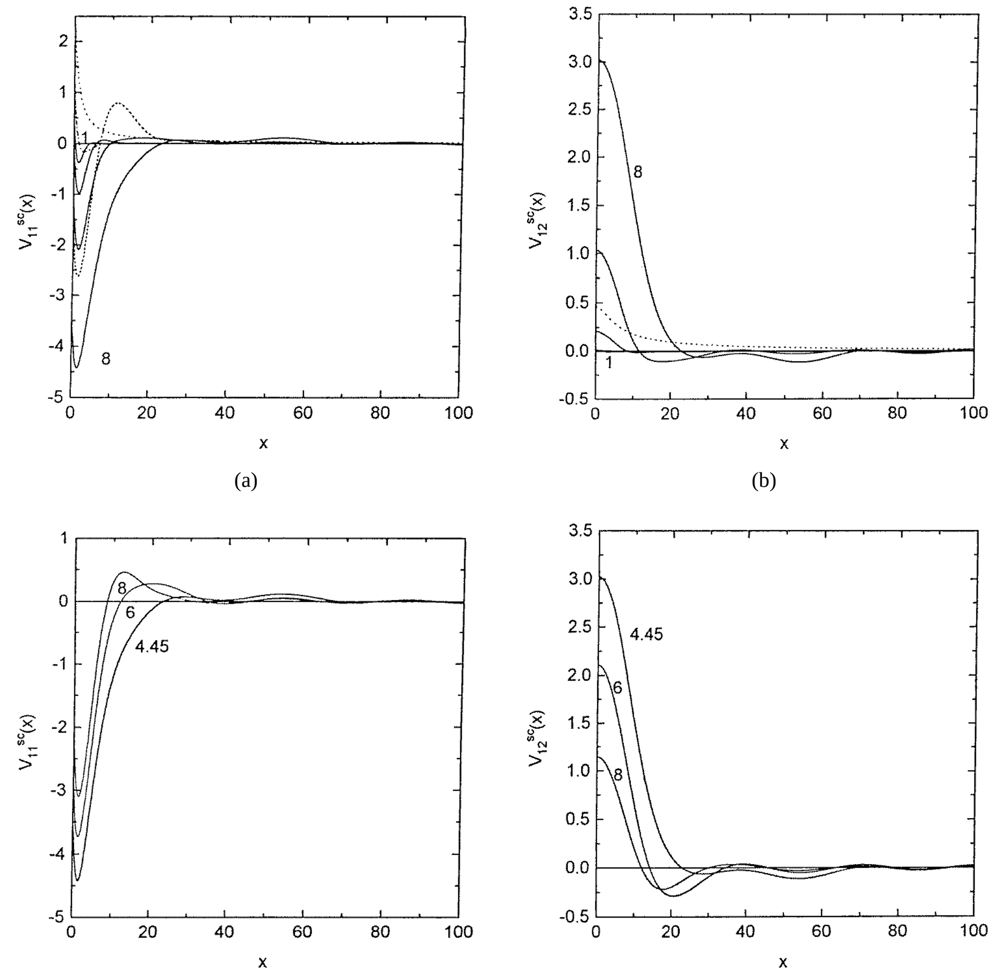


Figure 5. The intrawire and interwire pair-correlation functions $g_{11}(x)$ and $g_{12}(x)$. (a) $g_{11}(x)$; (b) $g_{12}(x)$; system parameters are the same as in figure 1. Labels represent the r_s-parameter. The dash–dot line is $g(x)$ for the single wire at $r_s = 8$. (c) $g_{11}(x)$; (d) $g_{12}(x)$; system parameters are the same as in figure 3. Labels represent the wire spacing d. x is in units of $(\pi/(4r_s a_0))^{-1}$.

Unscreened potential (dotted line), screening in a single wire (dashed line) and the result of the full RPA ($G_{11}(q) = G_{12}(q) = 0$) calculation (dash–dot line) are also plotted for comparison for density $r_s = 8$. Note, that the screened potential, unlike its unscreened counterpart, exhibits an attractive minimum with its magnitude being strongly enhanced over the full RPA by the many-body correlations. For example, at $r_s = 8$, the magnitude of the minimum in $V_{11}^{sc}(x)$ is about 20 times the result of the full RPA for $R_0 = 2$ and $d = 4.45$. This large difference in screening with the RPA indicates the extent of importance of the many-body correlations beyond the RPA. The comparison with the single-wire curve shows that the magnitude of the negative minimum is enhanced due to the presence of the second wire.

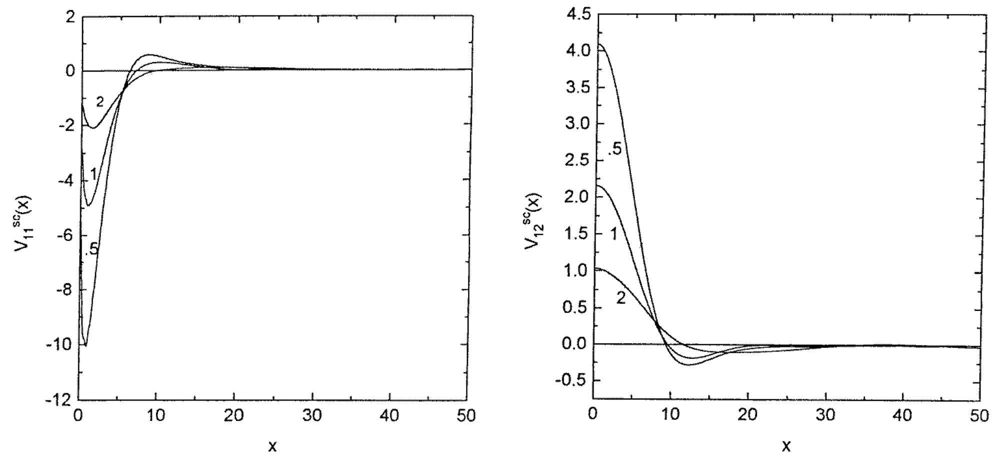
The dependence of the screened potentials on the separation between the wires d is shown in figures 6(c) and 6(d). Figures 6(e) and 6(f) illustrate the dependence of screening on the



(c) (d)

Figure 6. The intrawire and interwire static screened pair potentials $V_{11}^{sc}(x)$ and $V_{12}^{sc}(x)$. (a) $V_{11}^{sc}(x)$; (b) $V_{12}^{sc}(x)$; system parameters are the same as in figure 1. Labels represent the r_s-parameter. Dotted, dashed and dash–dot curves represent, respectively, the unscreened potential, screening in a single wire and the result of the full RPA at $r_s = 8$. Curves for $V_{11}^{sc}(x)$ from top to bottom correspond to $r_s = 1$, 3, 5 and 8. Curves for $V_{12}(x)$ from top to bottom correspond to $r_s = 8$, 5, 3 and 1. (c) $V_{11}^{sc}(x)$; (d) $V_{12}^{sc}(x)$; system parameters are the same as in figure 3. Labels represent the wire spacing d. (e) $V_{11}^{sc}(c)$; (f) $V_{12}^{sc}(x)$; system parameters the same as in figure 4. Labels represent the wire radius R_0.

wire size R_0. It can be noticed that decreasing the separation between wires, or making them narrow, both lead to qualitatively similar effects on the screening. There is a build-up in screening on decreasing both d and R_0 and a notable feature is that the screening is relatively more sensitive to R_0 than to d. Further, we have found for extremely narrow wires that $V_{11}^{sc}(x)$ becomes strongly attractive. For example, the depth of the minimum in V_{11}^{sc} is about -22 Ryd for $R_0 = 0.1$; $r_s = 1$ and $d = 0.3$. In the RPA ($G_{12}(q) = 0$) this magnitude reduces to -10 Ryd. Setting both $G_{11}(q) = G_{12}(q) = 0$ makes the minimum even shallower and it is about -2.5 Ryd. Thus, we see that though V_{11}^{sc} is already negative in the RPA, it is greatly



(e) (f)

Figure 6. (Continued)

enhanced due to the presence of interwire correlations and this effect becomes even more pronounced in the limit of narrow wires. In these situations, the system may become unstable against the formation of the bound pairs of bosons in each wire.

4. Charge-density-wave instability

Charge-density-wave instability corresponds to the relative accumulation of charge at some length scale, say q_c, in the system and if it happens, this can be checked by looking for a divergence in the static ($\omega = 0$) density susceptibility of the system. Diagonalizing the static density response matrix, the diagonal components of the susceptibility defined by $\chi(q) = -\chi(q, 0)$, are obtained as

$$\chi_{\pm}(q) = \frac{2/e^2}{r_s q^2 + f_{11}(q)(1 - G_{11}(q)) \pm f_{12}(q)(1 - G_{12}(q))}. \tag{17}$$

The wavevector q_c for the CDW ground state, if it exists, can be obtained by setting the denominator of equation (15) equal to zero, i.e.,

$$r_s q_c^2 + f_{11}(q_c)(1 - G_{11}(q_c)) \pm f_{12}(q_c)(1 - G_{12}(q_c)) = 0. \tag{18}$$

Equation (18) cannot be solved analytically for q_c. However, it becomes quite evident at first instance that it is the out-of-phase component of susceptibility which can diverge and the possibility for divergence will depend crucially on the behaviour of both the intra- and interwire local fields. Further, the divergence can never occur in the full RPA ($G_{11}(q) = G_{12}(q) = 0$) since equation (18) then can only be satisfied for imaginary q_c.

We attempt a numerical solution of equation (18) by using the self-consistent values of $G_{11}(q)$ and $G_{12}(q)$. We confine our investigation to the density range $1 \leqslant r_s \leqslant 8$ and to the range of wire spacing and size so that it remains physically justified to use the STLS approximation for dealing with the correlations. Interestingly enough, we do not find any acceptable solution of equation (18). This, in turn, rules out the possibility for the existence of a CDW ground state in the two coupled charged Bose quantum wires.

At this stage, it becomes particularly important to compare our results with the coupled electron quantum wire system. The two systems differ in the statistics obeyed by particles.

The CDW instability for the two coupled electron wires was first investigated by Gold [7] and, subsequently, by Wang and Ruden [8]. Gold predicted the instability in the long-wavelength limit, while Wang and Ruden showed in addition to the long-wavelength instability the presence of a CDW instability at $q/q_F = 2$. Wang and Ruden treated the intrawire correlations within the STLS approximation, while Gold used the Hubbard approximation. The effect of interwire correlations was neglected in both the calculations. Further, it was assumed that the intrawire correlations are not affected by the presence of the second wire. In our study, however, we have included both the intra- and interwire correlations on the same footing within the completely self-consistent STLS approach. The increasing importance of interwire correlations in the close proximity of two wires has already been demonstrated in figure 3. It is important to report here that using $G_{12}(q) = 0$ in equation (17) also gives a divergence in $\chi_-(q)$ below a critical separation d_c parallel to what is observed for the coupled electron wires. d_c is found to depend upon both r_s and R_0. Thus, we may arrive at the conclusion that the appearance of the CDW instability is an artifact of the neglect of interwire correlation effects. We also believe that the CDW instability predicted for the coupled electron wires may disappear, as in the case of coupled electron layers [6], if the interwire correlations are treated properly.

Further, though it is not possible to solve equation (18) analytically for q_c, we have obtained an important relation for the existence of long-wavelength instability. Using the long-wavelength results of form factors, equations (5) and (6), and of local fields, equation (13), q_c in the long-wavelength limit is obtained to be

$$q_c = \left\{ \frac{1}{r_s} \left[4 \ln(R_0/d) - 73/30 + (\gamma_{11} - \gamma_{12}) \right] \right\}^{1/2} \tag{19}$$

where $\gamma_{11} = G_{11}(q) f_{11}(q)$ and $\gamma_{12} = G_{12}(q) f_{12}(q)$. Both γ_{11} and γ_{12} are independent of q. Now, for the long-wavelength instability to occur the term inside the square brackets in the above equation must be greater than zero and in particular, for an instability at $q_c \approx 0$, we must have

$$\frac{d}{R_0} = \exp\left\{ \frac{1}{4} \left[(\gamma_{11} - \gamma_{12}) - 73/30 \right] \right\}. \tag{20}$$

The above condition can only be checked numerically since the R.H.S. is a constant depending upon r_s, d and R_0. We have found that this condition is not satisfied for the range of system parameters where the use of the STLS approximation is reliable. We therefore also rule out the possibility of the long-wavelength instability in a coupled charged Bose quantum wire system. Relation (20) is equally valid for two coupled electron wires and, hence, can be used to check the prediction of Gold for the existence of long-wavelength instability.

5. Conclusions

In conclusion, we have seen that the nature of the boson ground state is highly sensitive to the behaviour of interwire correlations. Neglecting them or their d-dependence causes the system to become unstable against transition into the charge-density-wave ground state below a critical layer spacing. However, the instability disappears completely with the inclusion of interwire correlation effects. This is an important finding of our study. We find by comparing our results with the similar studies on two coupled electron quantum wires that the CDW instability encountered there may also be the result of the neglect of interwire correlations.

Finally, it may be mentioned that at present we do not have any computer simulation data available to assess the accuracy of our results. However, it is well known that the STLS approximation used in this paper has been found to provide reasonable estimates of the ground-state properties in higher dimensions (i.e., 2D [12] and 3D [11]) for both the electron and charged Bose systems provided the density is not too low ($r_s \leqslant 5$). Therefore, we do not find any *a priori* reason to disbelieve the accuracy of STLS as applied to the system of two coupled Q1D charged Bose wires. Moreover, the same approximation has been used for studying the single (for example, see [14]) and coupled Q1D electron wires [15]. In addition to the coupled wires, the simulation study is also lacking for the coupled layers. Our prediction for the non-existence of CDW instability is in qualitative agreement with the result of Liu *et al* [6] for coupled layers. But, in the absence of the simulation or laboratory experiments it is difficult to make a final conclusion regarding the CDW instability. It is hoped that our present study will stimulate further work on this problem, in particular the simulation of the ground-state properties of the Q1D model.

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